MAT123

Graph Sine, Cosine

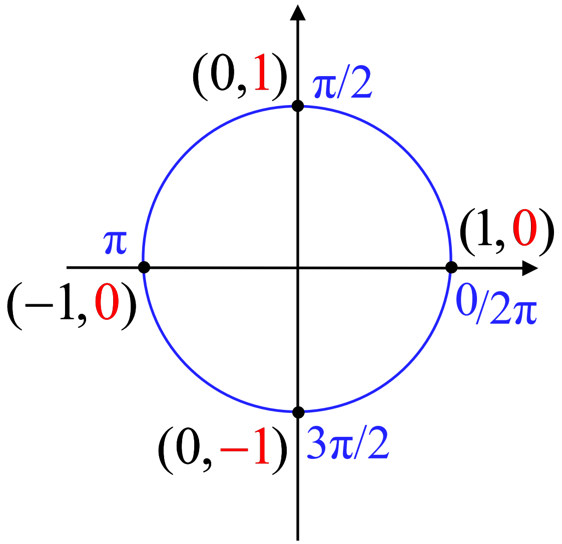
Periodic Functions

This model has a graph that oscillates, resembling a wave.
The *y*-values repeat in a pattern called a **period**.

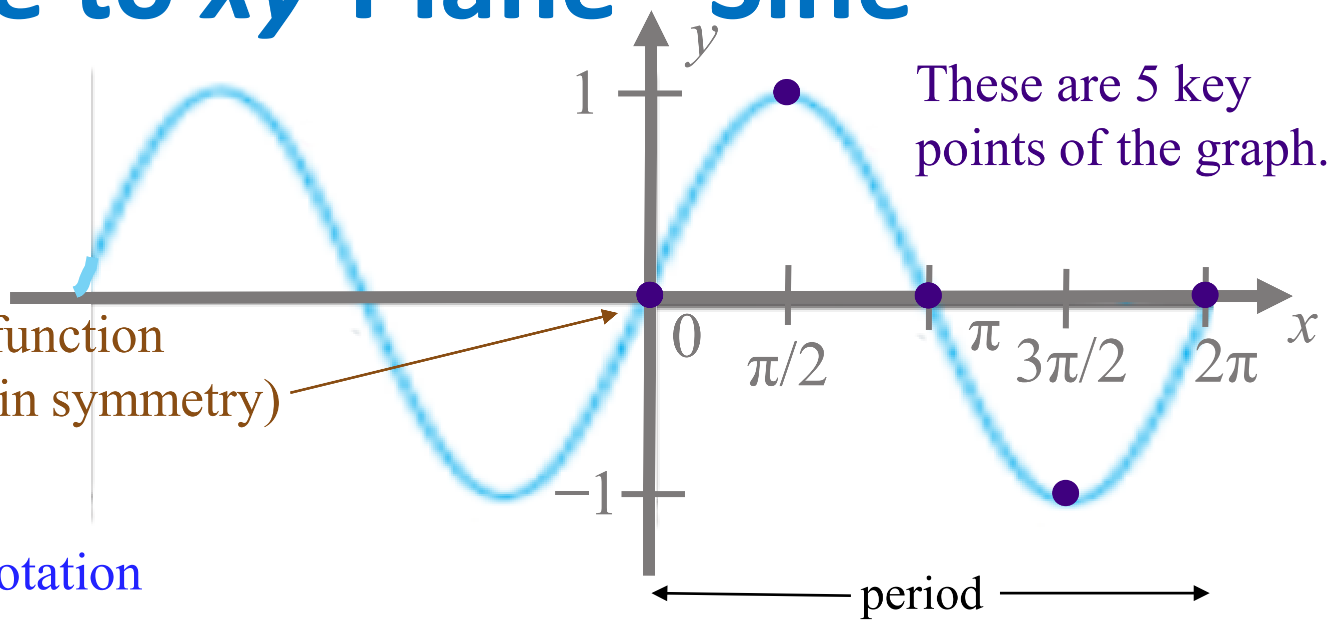
Real World Example:
tides

From Unit Circle to *xy*-Plane - Sine

Pertinent ordered pairs of sine graph:



plot y-coordinates
based on angle of rotation



$$y = \sin x$$

result, “y”

angle or θ

$$f(\theta) = \sin\theta$$

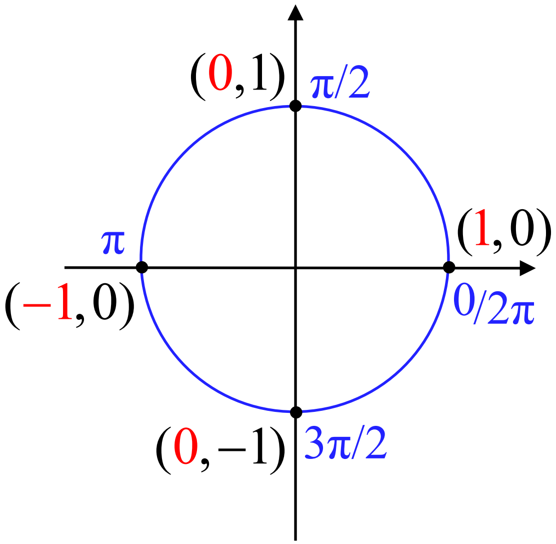
domain: $(-\infty, \infty)$ (infinitely many periods)

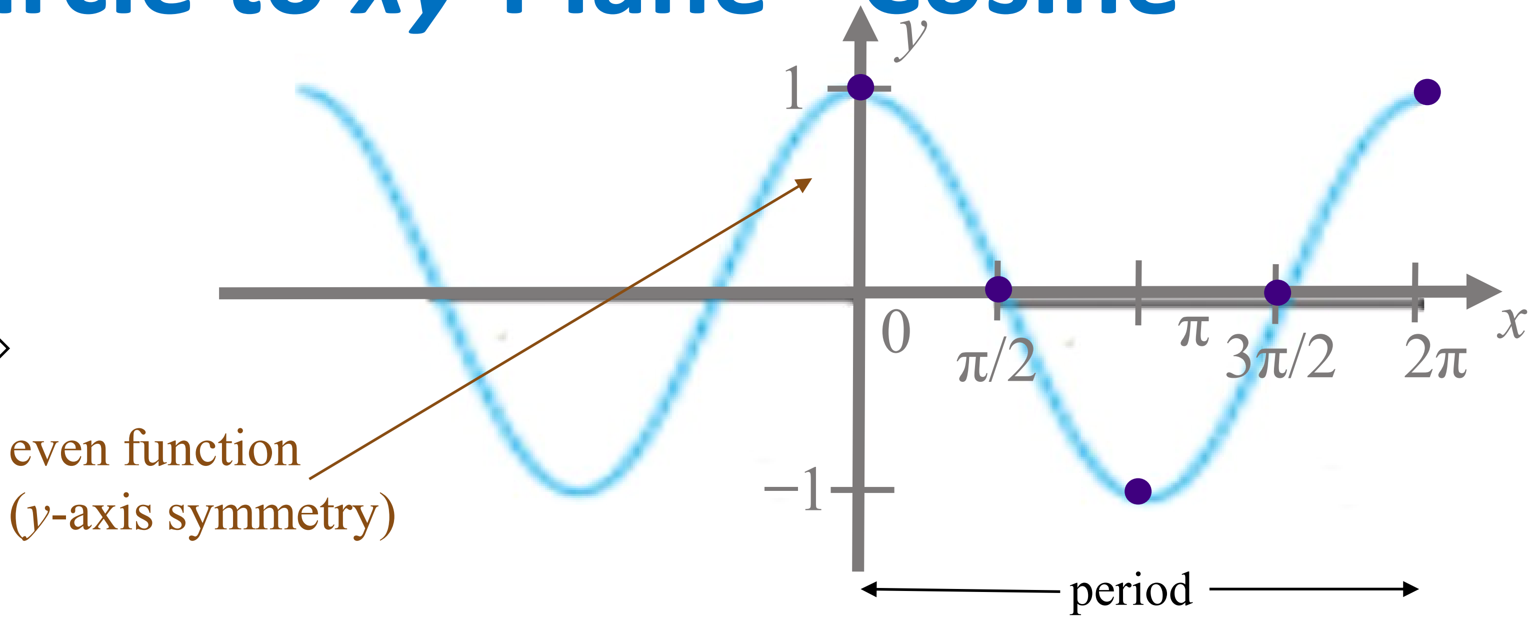
range: $[-1, 1]$

One period of $y = \sin x$ is 2π

From Unit Circle to *xy*-Plane - Cosine

Cosine uses the x-coordinates:





$$y = \cos x$$

or

$$f(\theta) = \cos\theta$$

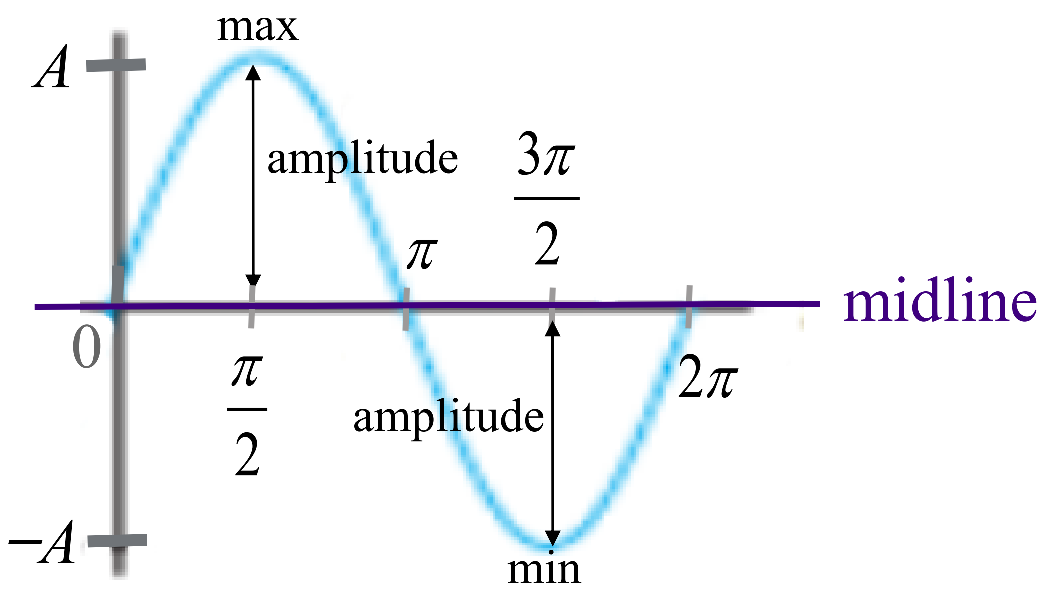
domain: $(-\infty, \infty)$

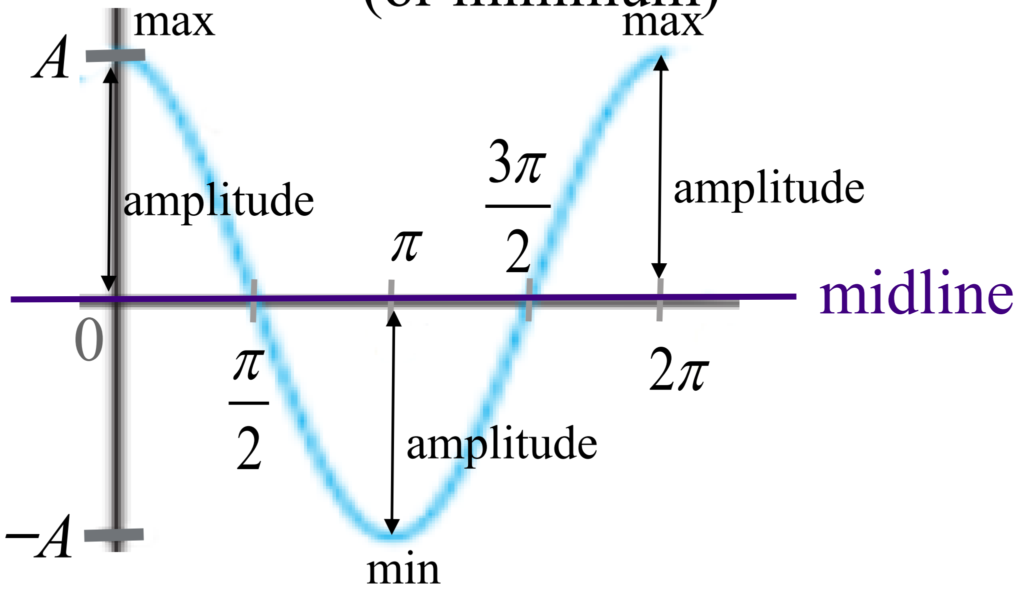
range: $[-1, 1]$

One period of $y = \cos x$ is 2π

Sine/Cosine – Vertical Stretch

Amplitude (vertical stretch) is the distance between the midline and the maximum y-value (or minimum)





$y = A\sin x$

in $y = \sin x$, $A = 1$

$y = A\cos x$

$y = \cos x : A = 1$

Can tell amplitude of function from $|A|$.

Amplitude Examples

ex. Graph one period of $y = 2\sin x$.

ex. Graph one period of $y = \frac{1}{2}\sin x$.

*

ex. Graph one period of $y = -2\sin x$.

note: amplitude is $|-2| = 2$

reflection about x-axis

Sine Graph w Varied Amplitude – Do

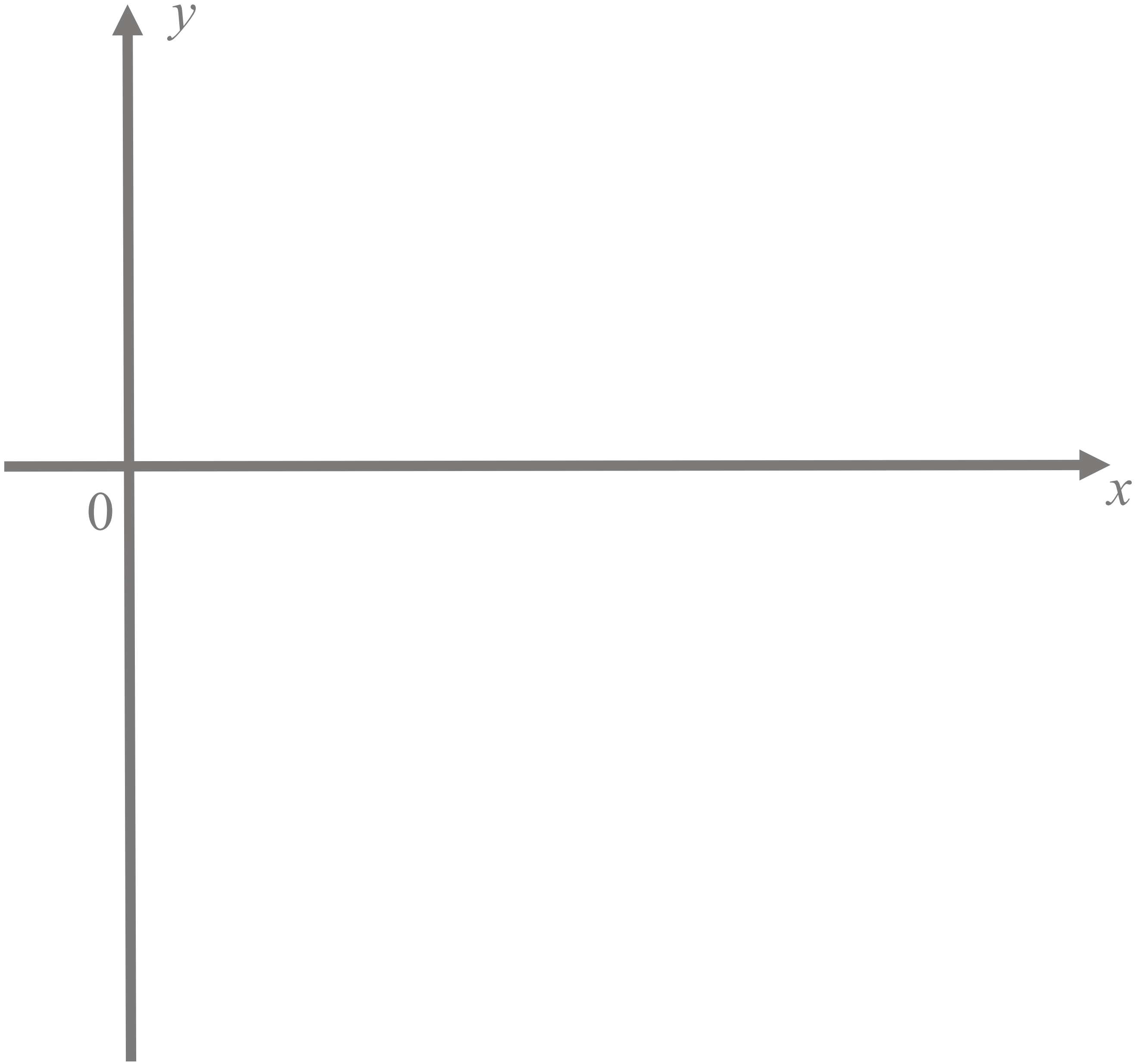
Do: Graph one period of $y = 4\sin x$.

Identify the 5 key points.

Identify domain and range of the period. (*in interval notation*)

domain: $[0, 2\pi]$

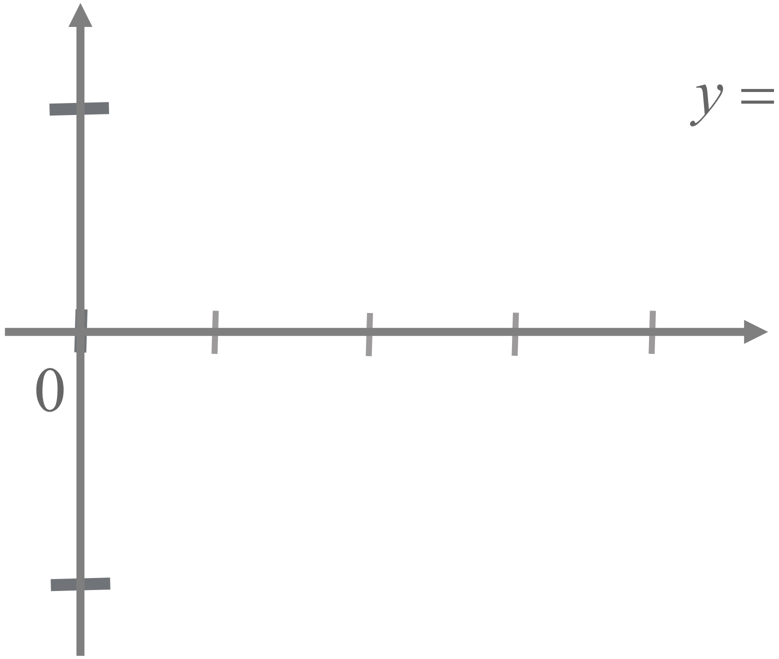
range: $[-4, 4]$



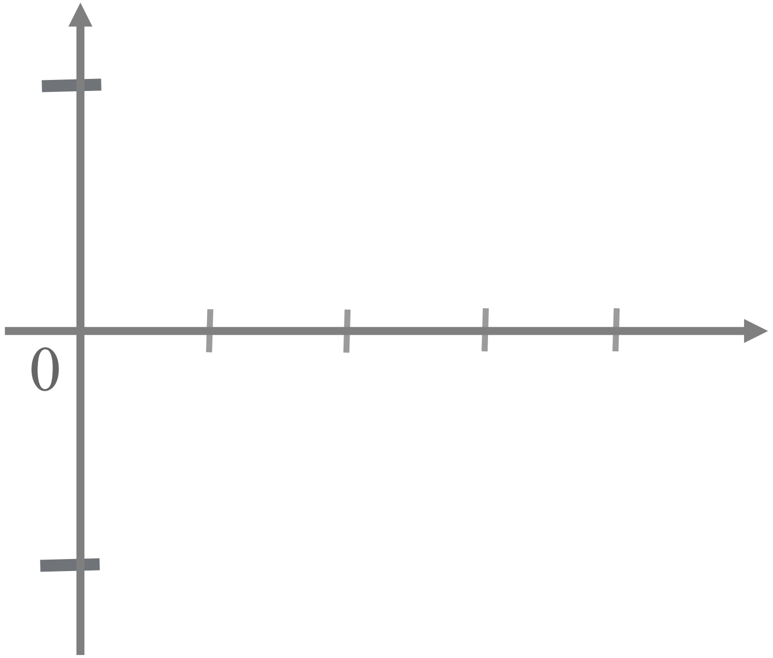
Sine/Cosine – Period Change

Do: sketch both

$y = \sin x$

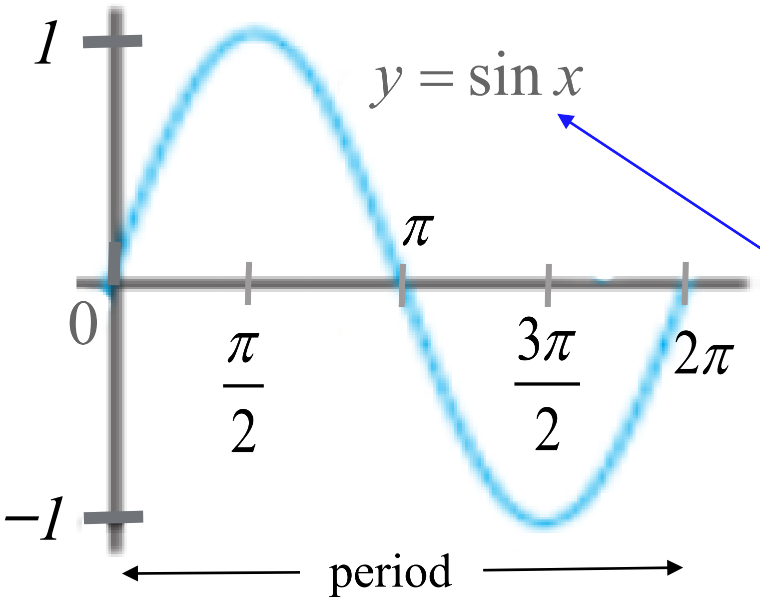


$y = \cos x$

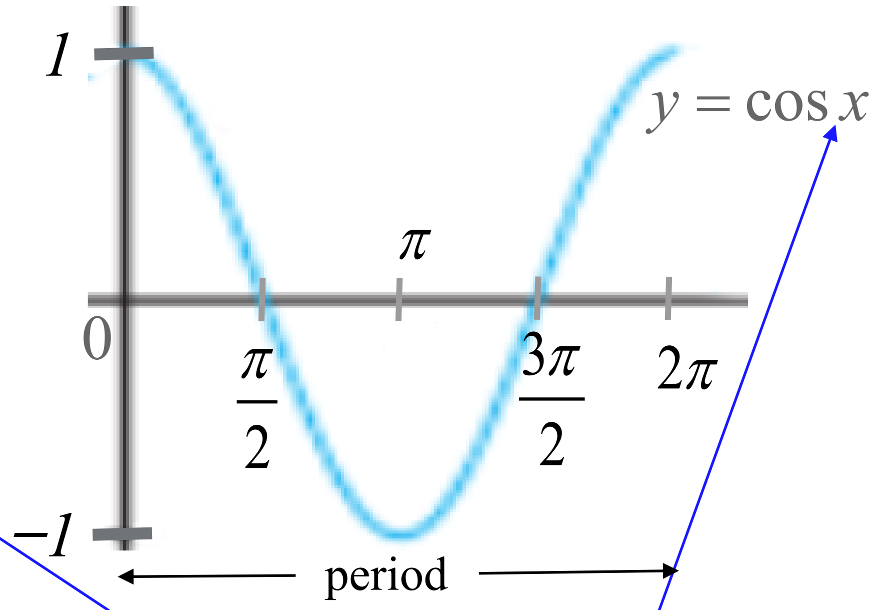


Sine/Cosine – Period Change

$y = A\sin(Bx)$



$y = A\cos(Bx)$



Calculate period using $\dfrac{2\pi}{B}$

in these examples, $B = 1$

therefore period remains 2π

Varied Period Examples

Graph one period of $y = 3\cos(2x)$.

1. draw shape w amplitude
2. determine period
3. calculate 5 key x-values

Graph one period of $y = 4\cos\left(\frac{x}{2}\right)$.

Graph one period of $y = \cos(8\pi x)$.

Amplitude is $|A|$

Calculate period using $\frac{2\pi}{B}$

Sine Graph w Transformations – Do

Do: Graph one period of $y = -2\sin(\pi x)$.

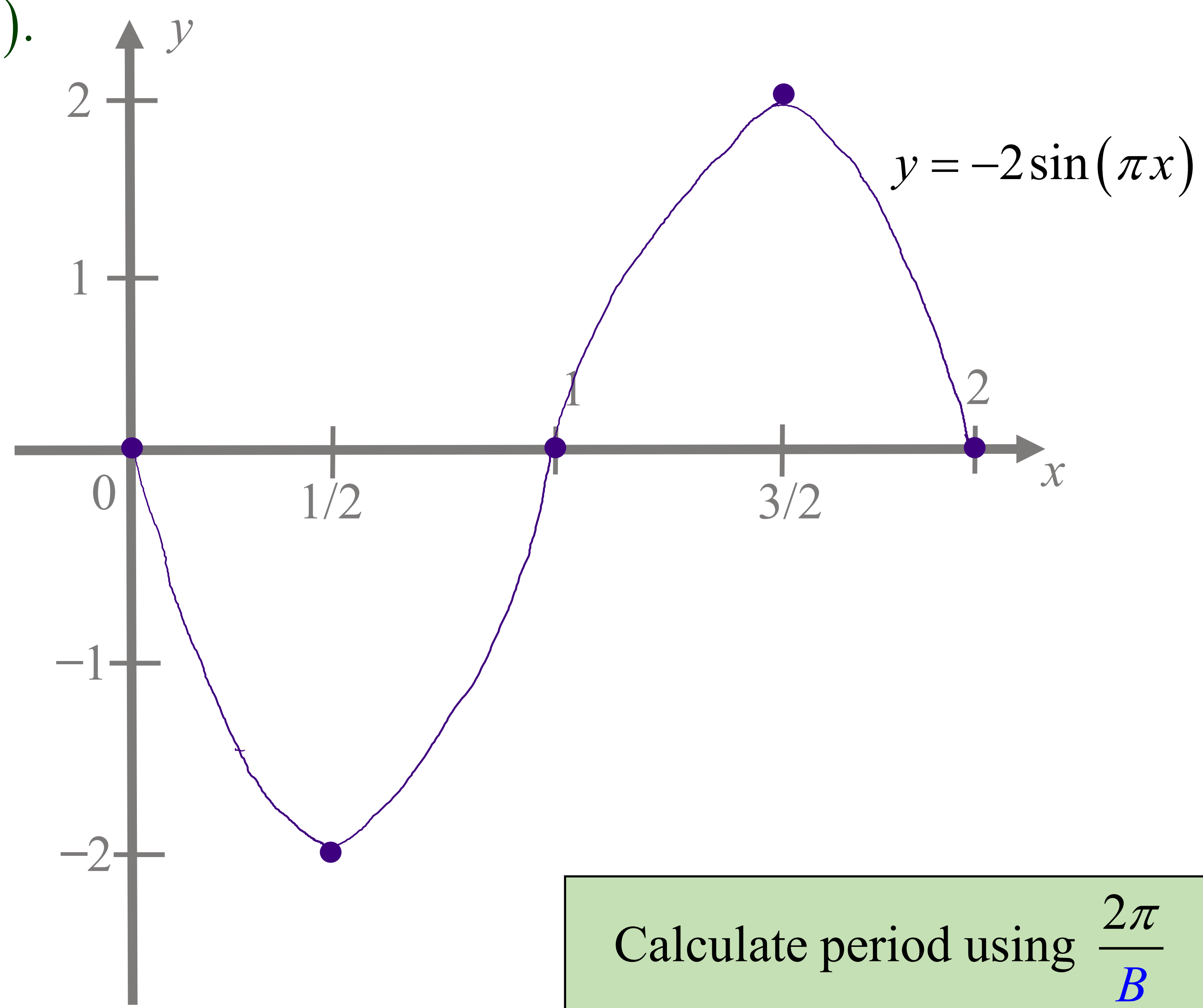
Label the 5 key points.

State amplitude.

$|-2| = 2$

State period.

$\frac{2\cancel{\pi}}{\cancel{\pi}} = 2$

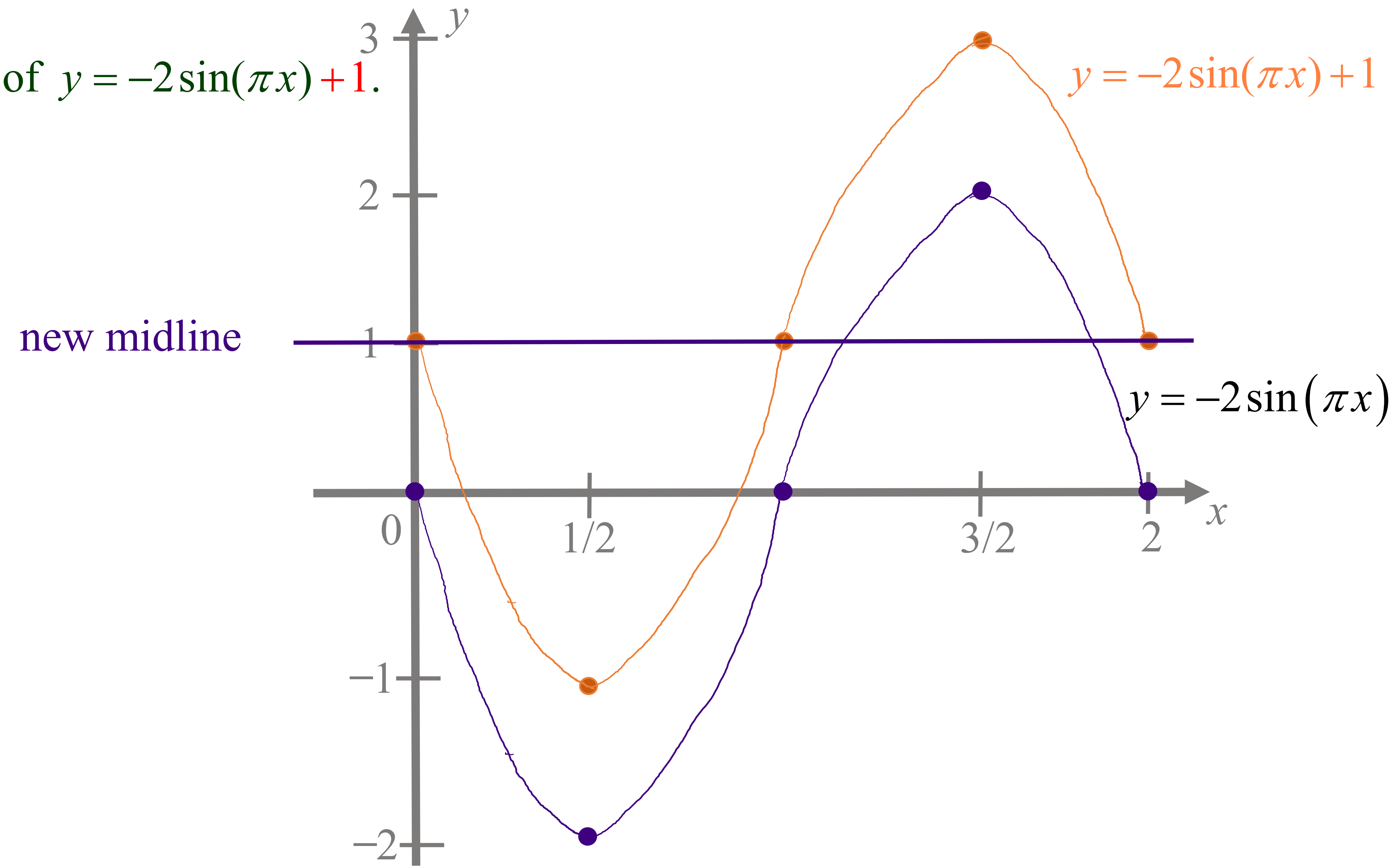


Calculate period using $\frac{2\pi}{B}$

Sine/Cosine – Vertical Shift

$y = A\sin(Bx) + D$ ⟵ shifts graph up/down D units

ex. Graph one period of $y = -2\sin(\pi x) + 1$.



Sine/Cosine – Horizontal or Phase Shift

$$y = A\sin(Bx - C) + D$$

moves graph left/right

Calculate phase shift using $\dfrac{C}{B}$

Do: Graph one period of $y = 4\sin(2x)$.

period is π
amplitude is $|4| = 4$

5 Key Points

pre-shift	w/shift
0	
$\frac{\pi}{4}$	
$\frac{\pi}{2}$	
$\frac{3\pi}{4}$	
π	

ex: Graph one period of $y = 4\sin\left(2x - \frac{2\pi}{3}\right)$.

phase shift: $\dfrac{C}{B} = \dfrac{\frac{2\pi}{3}}{2} = \dfrac{\cancel{2}\pi}{3} \cdot \dfrac{1}{\cancel{2}} = \dfrac{\pi}{3}$

domain of period:
$\left[\frac{\pi}{3}, \frac{4\pi}{3}\right]$

Sine/Cosine – Horizontal or Phase Shift - Do

Do: Graph one period of $y = 3\sin(2x - \pi)$.

Do: Graph one period of $y = \frac{1}{2}\cos\left(4x + \frac{\pi}{2}\right)$.

Calculate phase shift using $\frac{C}{B}$

Alternate Notation

You may see *Bx*–*C* in a factored form:

$$\text{ex: } y = 4\sin\left(2x - \frac{2\pi}{3}\right) \quad \text{versus} \quad \text{ex: } y = 4\sin\left[\underbrace{2}_{B}\left(x - \overset{C}{\frac{\pi}{3}}\right)\right]$$

get phase shift by looking at this value ONLY

… or you could distribute the 2 to make it look like the familiar format